Kerr Comb Generation in a Whispering Gallery Mode Microcavity: The Effect of Mode Coupling

Takasumi Tanabe,
Takumi Kato, Ryo Suzuki, and Shun Fujii

takasumi@elec.keio.ac.jp

Department of Electronics and Electrical Engineering,
Keio University, Japan
Outline

1. Background & Motivation
2. Modelling & effect of mode crossing
3. Effect of CW/CCW mode coupling
4. Effect of TM/TE mode coupling
5. Longitudinal mode family coupling via Raman nonlinearity
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Kerr comb in microcavity system

Convert CW laser to ultrashort pulse train w/ >600 GHz repetition rate

- Laser Diode
- CW input
- SiO₂ WGM μ-cavity
- Pulsed output
- Kerr comb generation w/ strong confinement of light

- Four-wave mixing
- Single wavelength

Optical power (dB)

Wavelength (nm)

Degenerate

Non-degenerate

Degenerate FWM

Non-degenerate FWM

Pump 500 mW

Ti:Sapphire laser based comb

large & expensive
Dispersion and coupled cavity for soliton

Soliton formation w/ negative dispersion

Coupled cavity for soliton generation (dark soliton)


A. Weiner: Nat. Photon. 9, 594-600 (2015)

Coupled cavity system is a new platform for the generation of Kerr comb
Dispersion control by using mode splitting

Cavity 1 (main cavity)

\[ \Delta \omega_- \quad \Delta \omega_+ \]

Coupling \( \kappa \)

Cavity 2 (auxiliary cavity)

\[ \Delta \omega_- \quad \Delta \omega_+ \]

Mode splitting

Anomalous: \( \Delta \omega_+ - \Delta \omega_- > 0 \)
Normal: \( \Delta \omega_+ - \Delta \omega_- < 0 \)

\[ \beta_2 = -\frac{n}{c} \cdot \frac{\Delta \omega_+ - \Delta \omega_-}{4\pi^2 \cdot \text{FSR}^2} \]

FSR modulated by coupling modes (local dispersion control)
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Simple Kerr comb simulation

\[ \frac{\partial \hat{A}_\mu}{\partial t} = \frac{Y_{A_0}}{2} \left( -1 + i \Omega_{A_\mu} \right) \hat{A}_\mu + \frac{Y_{A_0}}{2} \sum_{\alpha, \beta, \gamma} \delta_{\mu-(\alpha-\beta+\gamma)} \hat{A}_\alpha \hat{A}_\beta \hat{A}_\gamma + i \frac{\delta_{\omega_{A_\mu}-\omega_{B_\theta}}}{2} \frac{\kappa_{\theta}}{g_{AY_{B_0}}} \hat{B}_\theta + \frac{Y_{A_0}}{2} \delta_{\mu_0} f_0 \]

\[ \frac{\partial \hat{B}_\mu}{\partial t} = \frac{Y_{B_0}}{2} \left( -1 + i \Omega_{B_\mu} \right) \hat{B}_\mu + \frac{Y_{B_0}}{2} \sum_{\alpha, \beta, \gamma} \delta_{\mu-(\alpha-\beta+\gamma)} \hat{B}_\alpha \hat{B}_\beta \hat{B}_\gamma + \frac{\delta_{\omega_{B_\mu}-\omega_{A_\theta}}}{2} \frac{\kappa_{\theta}}{g_{BY_{A_0}}} \hat{A}_\theta \]

2. Modelling the coupled cavity: Effect of mode crossing

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Cavity A

Cavity B

Normalized power (a.u.)

Normalized power (a.u.)

\( \kappa_{\mu} / \gamma_{\theta} = 1.2 \)

\( \kappa_{\mu} / \gamma_{\theta} = 1.2 \)

\( \kappa = 0 \)

\( \kappa = \gamma_{\theta} / 2 \)

\( \kappa = \gamma_{\theta} / 2 \)

\( \gamma_{\theta} / 2 \)

\( \gamma_{\theta} / 2 \)
Dark soliton in normal dispersion cavity

Mode crossing is needed to initialize soliton formation

\[ Q_{cav} = 8.6 \times 10^5 \]
\[ r = 100 \mu m \]
\[ \lambda_0 = 1545.4 \text{ nm} \]
\[ n = 2.46 \]
\[ \kappa = 1 \text{ GHz} \]
\[ P_{in} = 800 \text{ mW} \]
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Strong coupling between CW/CCW with small cavity

- Coupling rate: $\kappa$
- Decay rate: $\gamma = (\gamma_0 + \gamma_{\text{taper}})$

$\kappa = 85$ MHz
$\gamma = 6.4$ MHz
$Q_{\text{load}} = 3.1 \times 10^7$

Resonance splitting (Freq. domain)

Energy oscillation (Time domain)

$\tau_{\text{load}} = 15.9$ ns
$\kappa/2\pi = 76$ MHz
$\gamma/2\pi = 10$ MHz
$\Gamma \approx 13$

Effect of CW/CCW coupling to Kerr comb

Experimental setup

CW/CCW comb & coupling strength

3. CW/CCW mode coupling

Photonic Structure Group, Keio University
3. CW/CCW mode coupling

Effect of CW/CCW coupling: RF measurement

- RF noise measurement

- RF spectrum

Each line is filtered out and the RF spectrum is measured.

No mode splitting effect is observed (Only the effect of cavity-opto mechanics is present).
Numerical analysis of CW/CCW coupling based on nonlinear CMT equations

\[
\frac{\partial \tilde{A}_\mu}{\partial t} = \frac{\gamma_{A_0}}{2} \left( -1 + i \Omega_{A_\mu} \right) \tilde{A}_\mu + \frac{\gamma_{A_0}}{2} \cdot i \sum_{\alpha,\beta,\gamma} \delta_{\mu-(\alpha-\beta+\gamma)} \tilde{A}_\alpha \tilde{A}_\beta \tilde{A}_\gamma + \frac{\gamma_{B_0}}{2} \left( 1 + i \Omega_{B_\mu} \right) \tilde{B}_\mu + \frac{\gamma_{B_0}}{2} \cdot i \sum_{\alpha,\beta,\gamma} \delta_{\mu-(\alpha-\beta+\gamma)} \tilde{B}_\alpha \tilde{B}_\beta \tilde{B}_\gamma
\]

\[
\frac{\partial \tilde{B}_\mu}{\partial t} = -\frac{\gamma_{B_0}}{2} \left( 1 - i \Omega_{B_\mu} \right) \tilde{B}_\mu + \frac{\gamma_{A_0}}{2} \cdot i \sum_{\alpha,\beta,\gamma} \delta_{\mu-(\alpha-\beta+\gamma)} \tilde{A}_\alpha \tilde{A}_\beta \tilde{A}_\gamma + \frac{\gamma_{B_0}}{2} \left( 1 + i \Omega_{B_\mu} \right) \tilde{B}_\mu + \frac{\gamma_{A_0}}{2} \cdot i \sum_{\alpha,\beta,\gamma} \delta_{\mu-(\alpha-\beta+\gamma)} \tilde{B}_\alpha \tilde{B}_\beta \tilde{B}_\gamma
\]

Normalized nonlinear coupled mode equation

\[
A_\mu = \sqrt{2g_A/\gamma_{A_0}} A_\mu e^{i(\omega_{A_\mu} - \omega_{\text{in}} - \mu D_{1A})t}, \quad \Omega_{A_\mu} = 2 \left( \omega_{A_\mu} - \omega_{\text{in}} - \mu D_{1A} \right)/\gamma_{A_0}, \quad f_0 = \sqrt{8g_A \gamma_{\text{ext}}/\kappa_{A_0}^3 A_{\text{in}}}
\]

\[
B_\mu = \sqrt{2g_B/\kappa_{B_0}} B_\mu e^{i(\omega_{B_\mu} - \omega_{\text{in}} - \mu D_{1B})t}, \quad \Omega_{B_\mu} = 2 \left( \omega_{B_\mu} - \omega_{\text{in}} - \mu D_{1B} \right)/\gamma_{B_0}
\]
Soliton formation in CW/CCW mode coupled system

3. CW/CCW mode coupling

Soliton formed in CW only
Soliton formation in CW/CCW mode coupled system

Coupling $\Gamma = 0 \sim 5.0$

Soliton formed in CW only

Input power $P_{in} = 300 \text{ mW}$

Coupling $\Gamma = 1$

No noise appears when both CW/CCW modes are in soliton state
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Influence of XPM on Kerr comb: simulation

- Lugiato-Lefever equations w/ TE/TM coupling

\[
\frac{t_R}{\partial r} \frac{\partial E_{TE}}{\partial r} = \left( -\alpha_{TE} - i\delta_{TE} + iL \sum_{k=2}^{\infty} \frac{\beta_{k,TE}}{k!} \left( i \frac{\partial}{\partial T} \right)^k \right) E_{TE} + i\gamma_{TE}L(|E_{TE}|^2 + PB|E_{TM}|^2)E_{TE} + \sqrt{\kappa_{TE}} S_{TE}
\]

\[
\frac{t_R}{\partial r} \frac{\partial E_{TM}}{\partial r} = \left( -\alpha_{TM} - i\delta_{TM} + dLi \frac{\partial}{\partial T} + iL \sum_{k=2}^{\infty} \frac{\beta_{k,TM}}{k!} \left( i \frac{\partial}{\partial T} \right)^k \right) E_{TM} + i\gamma_{TM}L(|E_{TM}|^2 + PB|E_{TE}|^2)E_{TM} + \sqrt{\kappa_{TM}} S_{TM}
\]

1. Mode overlapping \((B)\) is perfect.
   → \(B\) should be changed. \((0\sim1.0)\)

2. Group velocity mismatch \((d)\) is zero.
   → \(d\) has to be calculated.

3. Detuning & cavity length \((L)\) are same.
   → different detuning has to be set.

4. Linear coupling is zero.
   → EPFL members are interested in linear coupling condition.
Influence of XPM on Kerr comb: simulation

- Dual comb generation from dual input...

TE Input power: 100 mW
TM Input power: 50 mW

(1) (2) (3)

(1a) (1b) (2a) (2b) (3a) (3b)
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Different longitudinal mode coupling (experiment)

Resonator 1
- $\lambda_{\text{pump}} = 1557.3$ nm

Resonator 2
- $\lambda_{\text{pump}} = 1557.5$ nm

Same longitudinal mode family is generated

Different longitudinal mode family via Raman

Pair No. 5

Different mode is generated w/ 180 GHz offset
Coupled Lugiato-Lefever Equations

**Lugiato-Lefever equations with Raman interaction**

**Mode 1 (pump mode)**

\[
t_R \frac{\partial E_p}{\partial t} = \left\{ -\frac{\alpha_p}{2} - \frac{\kappa_p}{2} - i\delta_p + iL \sum_{k \geq 2} \frac{\beta_p^{(k)}}{k!} \left( -i \frac{\partial}{\partial t} \right)^k \right\} E_p + iLN_p + \sqrt{\kappa_p}S_{in}
\]

\[
N_p = (1 - f_R) \left( \gamma_p |E_p|^2 + 2\Gamma_p |E_s|^2 \right) E_p + f_R \begin{cases} 
\text{SPM} & \gamma_p E_p \int_{-\infty}^{\infty} h_R(t')|E_p(t - t')|^2 dt' \\
\text{XPM} & +\Gamma_p E_p \int_{-\infty}^{\infty} h_R(t')|E_s(t - t')|^2 dt' + \Gamma_p E_s \int_{-\infty}^{\infty} h_R(t')E_p(t - t')E_s^*(t - t')dt'
\end{cases}
\]

**Mode 2 (Raman mode)**

\[
t_R \frac{\partial E_s}{\partial t} = \left\{ -\frac{\alpha_s}{2} - \frac{\kappa_s}{2} - iL(\beta_s^{(1)} - \beta_p^{(1)}) \left( -i \frac{\partial}{\partial t} \right) + iL \sum_{k \geq 2} \frac{\beta_s^{(k)}}{k!} \left( -i \frac{\partial}{\partial t} \right)^k \right\} E_s + iLN_s
\]

\[
N_s = (1 - f_R) \left( \gamma_s |E_s|^2 + 2\Gamma_s |E_p|^2 \right) E_s + f_R \begin{cases} 
\text{SPM} & \gamma_s E_s \int_{-\infty}^{\infty} h_R(t')|E_s(t - t')|^2 dt' \\
\text{XPM} & +\Gamma_s E_s \int_{-\infty}^{\infty} h_R(t')|E_p(t - t')|^2 dt' + \Gamma_s E_p \int_{-\infty}^{\infty} h_R(t')E_s(t - t')E_p^*(t - t')dt'
\end{cases}
\]
Spectrum and intra-cavity power

\[ Q = 2 \times 10^7 \]

Pump 100 mW

No mode2

\[ B = 0.6 \]

Pump power = 100 mW

\[ Q_1 = 2 \times 10^7 \]

\[ Q_2 = 6 \times 10^7 \]

\[ B = 0.6 \]

Pump 100 mW

\[ Q_1^{pump} = 2 \times 10^7 \]

\[ Q_e = 2 \times 10^8 \]

Pump power = 100 mW
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Summary

We investigate the effect of mode coupling

1. **Mode crossing**: Mode crossing needed to initiate dark soliton

2. **CW/CCW mode coupling**: Small noise present from CCW mode scattering, but this can be suppressed by choosing right parameters.

3. **TM/TE mode coupling**: Injection locking occurs which is a powerful method to achieve dual comb generation

4. **Nonlinear coupling between modes via Raman**: Mode coupling occurs from a low to high Q

Mode coupling enable rich function
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