Soliton trapping in a Kerr microresonator with orthogonally polarized dual-pumping

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Funding
Microcombs

Laser light having a comb-like spectrum, which is generated from a microresonator.

“Microcombs” or “Kerr combs”

Microresonator

- Compact size
- Low consumption energy
- Large mode spacing \((f_{\text{rep}} \approx 10-1000 \text{ GHz})\)

Applications
- Optical communications
- Dual-comb spectroscopy
- Dual-comb LiDAR
- Microwave oscillators
- Optical frequency synthesizers

“Frequency combs”

- Small mode spacing \((f_{\text{rep}} \approx 0.01-10 \text{ GHz})\)

Applications
- Optical communications
- Dual-comb spectroscopy
- Dual-comb LiDAR
- Microwave oscillators
- Optical frequency synthesizers

Microresonator

Ti:Sapphire laser

Fiber laser

Comb spectrum

\[
\begin{align*}
\text{Optical Power} & \quad 1^{\text{st}} \quad 2^{\text{nd}} \quad \ldots \quad \ldots \quad m^{\text{th}} \quad f_{\text{rep}} \\
\text{Frequency} & \quad f_{\text{CEO}} \quad \quad f(m) = f_{\text{CEO}} + m \cdot f_{\text{rep}}
\end{align*}
\]
Dual-comb applications: scan rate \( \Leftrightarrow \) difference of repetition frequencies

Microcombs have a potential to achieve fast scan rate due to high repetition frequencies

**LiDAR**
- Ultrafast ranging

**Spectroscopy**
- Absorption spectrum
- Dual-Comb Spectroscopy
- RF Domain

**CARS**
- Hexachlorobenzene
- Nitromethane

**Dual-comb generation in a single resonator**

**Advantages**
- Simple setup
- Both combs share the same resonator (common mechanical vibrations) and the feedback loops, which lead to mutual coherence.

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*Science 359, 887-891 (2018)*
*Science 354, 600-603 (2016)*
*Science 356, 1164-1168 (2017)*
*Nature 502, 355-358 (2013)*
Recently, some experimental demonstrations have been reported.

Left: Nat. Photonics 11, 560-564 (2017)
Interaction between two solitons in a microresonator has not been well understood. Here we focus on soliton trapping between orthogonally polarized solitons.

In previous research, soliton trapping has been observed experimentally via Raman effects with single-pumping.

In this work, we consider a system where two solitons are excited with dual-pumping having orthogonally polarizations.

In this work,
- Develop a simulation model based on coupled Lugiato-Lefever equations (LLEs), taking cross-phase modulation (XPM) and repetition difference terms into account.
- Calculate with generalized parameters to reveal trapping conditions
- Perform analysis of coupled solitons solutions.
Coupled Lugiato-Lefever equations (LLEs)

\[
\frac{\partial a}{\partial t} = -\frac{\kappa(a)}{2} a + i \Delta \omega_{0(a)} a + i \frac{D_{2(a)}}{2} \frac{\partial^2 a}{\partial \phi^2} + i g(a) (|a|^2 + \sigma |b|^2) a + \sqrt{\kappa_c(a)} s_{in(a)} + \frac{\Delta D_1}{2} \frac{\partial a}{\partial \phi}
\]

\[
\frac{\partial b}{\partial t} = -\frac{\kappa(b)}{2} b + i \Delta \omega_{0(b)} b + i \frac{D_{2(b)}}{2} \frac{\partial^2 b}{\partial \phi^2} + i g(b) (|b|^2 + \sigma |a|^2) b + \sqrt{\kappa_c(b)} s_{in(b)} - \frac{\Delta D_1}{2} \frac{\partial b}{\partial \phi}
\]

(loss) (detuning) (dispersion) (Kerr effects) (input) (repetition difference)

\( t \): time, \( \phi \): angular coordinate, \( a, b \): internal fields, \( \kappa \): resonator loss, \( \Delta \omega_0 \): pump detuning, \( D_2 \): second order dispersion, \( g \): nonlinear coefficient, \( \sigma \): XPM coefficient (\( \sigma = 2/3 \) for orthogonally polarizations), \( \kappa_c \): coupling rate, \( s_{in} \): input field, \( \Delta D_1 \): FSR (repetition frequency) difference

Dimensionless coupled LLEs  (Assuming \( \kappa = \kappa(a) = \kappa(b), g = g(a) = g(b) \))

\[
\frac{\partial u}{\partial \tau} = -(1 + i \alpha(u)) u + i \beta(u) \frac{\partial^2 u}{\partial \phi^2} + i (|u|^2 + \sigma |v|^2) u + F(u) + \gamma \frac{\partial u}{\partial \phi}
\]

\[
\frac{\partial v}{\partial \tau} = -(1 + i \alpha(v)) v + i \beta(v) \frac{\partial^2 v}{\partial \phi^2} + i (|v|^2 + \sigma |u|^2) v + F(v) - \gamma \frac{\partial v}{\partial \phi}
\]

\( \tau = \frac{1}{2} \kappa t, u = \sqrt{\frac{2g}{\kappa}} a, v = \sqrt{\frac{2g}{\kappa}} b, \alpha(\cdot) = -\frac{2\Delta \omega_{0(\cdot)}}{\kappa}, \beta(\cdot) = \frac{D_{2(\cdot)}}{\kappa}, \gamma = \frac{\Delta D_1}{\kappa}, F(\cdot) = \frac{2g \kappa_c(\cdot)}{\kappa} s_{in(\cdot)} \)

Relations, \( \alpha \): detuning, \( \beta \): second order dispersion, \( \gamma \): repetition difference, \( F \): input
Soliton trapping with dimensionless coupled LLEs

\[
\begin{align*}
\frac{\partial u}{\partial \tau} &= -(1 + i\alpha(u))u + i\beta(u)\frac{\partial^2 u}{\partial \phi^2} + i(|u|^2 + |v|^2)u + F(u) + \gamma \frac{\partial u}{\partial \phi} \\
\frac{\partial v}{\partial \tau} &= -(1 + i\alpha(v))v + i\beta(v)\frac{\partial^2 v}{\partial \phi^2} + i(|v|^2 + |u|^2)v + F(v) - \gamma \frac{\partial v}{\partial \phi}
\end{align*}
\]

\[\beta(\ast) = 0.01, \gamma = 0.3, F(\ast) = 4\]
\[\alpha \text{ is scanned}\]

Relations, \(\alpha\): detuning, \(\beta\): second order dispersion, \(\gamma\): repetition difference, \(F\): input
Trapping conditions as functions of $F$ and $\delta$

Group velocities are compensated with XPM

Center frequency shift:
\[ \Delta \omega = \frac{\Delta D_1}{2D_2} \times D_1 \]

Waveforms

Relations $\alpha$: detuning, $\beta$: second order dispersion, $\gamma$: repetition difference, $F$: input, $\delta = \gamma(2\beta)^{-0.5}$
Analysis of coupled solitons solutions

Dimensionless coupled LLEs

\[
\frac{\partial u}{\partial \tau} + i \alpha_u(u) - i \frac{1}{2} \frac{\partial^2 u}{\partial \theta^2} - i(|u|^2 + \sigma|v|^2)u - \delta \frac{\partial u}{\partial \theta} = F(u) - u
\]

\[
\frac{\partial v}{\partial \tau} + i \alpha_v(v) - i \frac{1}{2} \frac{\partial^2 v}{\partial \theta^2} - i(|v|^2 + \sigma|u|^2)v + \delta \frac{\partial v}{\partial \theta} = F(v) - v
\]

Ansatz of coupled solitons for perturbed Lagrangian approach

\[
u = B \text{sech}(\sqrt{1 + \sigma B \theta}) \exp(i \phi_0) \exp(i \delta \theta), \quad v = B \text{sech}(\sqrt{1 + \sigma B \theta}) \exp(i \phi_0) \exp(-i \delta \theta)
\]

Relations, \( \alpha \): detuning, \( \beta \): second order dispersion, \( \gamma \): repetition difference, \( F \): input, \( \delta = \gamma(2\beta)^{-0.5} \)

\[\theta = \frac{1}{\sqrt{2\beta}} \phi, \quad \delta = \frac{\gamma}{\sqrt{2\beta}}\]
Strong soliton supports weak soliton generation

\[ |u|^2 \text{ or } |v|^2 \]

\( \alpha = -2 \)
\( \alpha = 0 \)
\( \alpha = 5 \)
\( \alpha = 10 \)
\( \alpha = 16 \)
\( \alpha = 22 \)

Intracavity power

Spectrum

Waveform

F = 4
F = 3
Strong soliton supports weak soliton generation

Intracavity power

F = 4
F = 3

Spectrum Waveform

\[ \alpha = -2 \]
\[ \alpha = 0 \]
\[ \alpha = 5 \]
\[ \alpha = 10 \]
\[ \alpha = 16 \]
\[ \alpha = 22 \]
Summary

- Developed simulation model with coupled LLEs, which include XPM and repetition difference terms
- Calculated with generalized parameters to reveal trapping conditions
- Performed analysis of coupled solitons solutions

\[
\frac{\partial u}{\partial \tau} = -(1 + i\alpha(u))u + i\frac{1}{2} \frac{\partial^2 u}{\partial \theta^2} + i(|u|^2 + \sigma|v|^2)u + F(u) + \delta \frac{\partial u}{\partial \theta}
\]

\[
\frac{\partial v}{\partial \tau} = -(1 + i\alpha(v))v + i\frac{1}{2} \frac{\partial^2 v}{\partial \theta^2} + i(|v|^2 + \sigma|u|^2)v + F(v) - \delta \frac{\partial v}{\partial \theta}
\]