Kerr comb generation in a mode coupled system

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Tuesday 5 February, 2019 1:50PM – 2:15PM
Room 303 (South Level Three)
High-Q whispering-gallery mode microcavities

- **Silicon nitride**
  - Weiner group (Purdue)

- **Diamond**
  - Loncar group (Harvard)

- **Crystalline (CaF$_2$, MgF$_2$, etc)**
  - Kippenberg group (EPFL, Swiss), Makei group (OE Waves)

- **Silicon**
  - Gaeta group (Columbia)

- **AlN**
  - Tang group (Yale)

- **Silica**
  - Vahala group (Caltech)

- **AlGaAs**
  - Yvind group (DTU, Denmark)

**Q-factor**

\[ Q = \omega \times \frac{\text{stored energy}}{\text{power in/out}} \]

**Photon density**

\[ \propto \frac{Q}{V} \]
Third-harmonic generations in toroid microcavity

Kerr comb in microcavity system

Convert CW laser to ultrashort pulse train w/ > 800 GHz rep. rate

Ti:Sapphire laser based comb
large & expensive

Degenerate FWM
Non-degenerate FWM

Degenerate Non-degenerate

Fiber laser
Ti:S laser
Kerr comb

10 MHz 100 MHz 1 GHz 10 GHz 100 GHz 1 THz

Pump
Wavelength (nm)

Power (dBm)

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Required conditions for soliton formation

- Soliton in a μ-cavity
- Dispersion in a small toroid

Mode-lock
≈ Soliton

Nonlinear (Kerr) phase
≈ Anomalous dispersion

- Material dispersion
- Geometric dispersion

Dispersions in toroid microcavity ($r = 35 \mu m$)
Design the dispersion

Introduction


Mode profile

Displacement (geometric + material)

- Normal Dispersion
- Anomalous Dispersion

Over one octave

Mode profile

Dispersion (geometric + material)
Fabrication of CaF$_2$ WGM cavity w/ cutting

- **Precise machining process**
  

- **Computer controlled lathe cutting**
  

  ![](image)

  \[ R_{\text{rms}} = 3 \text{ nm} \]

  \[ Q = 3.0 \times 10^7 \]

  Preliminarily

  (2018/Nov.)

- CaF$_2$ can be smoothly cut in ductile mode cutting
Kerr comb in microcavity system

Silica toroid microcavity

- > 800 GHz

Silica rod microcavity

- 10~160GHz

Efficiency and the use of dark pulse (normal dispersion)

- Coupling is low with a bright pulse

Nonlinear (Kerr) phase

\[ \Delta f(t) \propto -n_2 \frac{\partial I}{\partial t} \]

\[ \approx \text{Normal dispersion} \]

X. Xue et al., Nat. Photon. 10 (2015)

CW pump

Pulse output
Kerr combs with mode coupling

Two modes coupling

CW/CCW mode coupling

TE/TM mode coupling

Dark pulse generation at normal dispersion

Effect of inherent coupling

Dual comb generation

Outline

**Kerr combs with mode coupling**

### Two modes coupling

- Continuous wave (input) vs. time
- ECDL
- Dark pulse (output)

### CW/CCW mode coupling

- Microcavity
- Mode coupling $\kappa$
- Mode-locked pulse
- Pump
- Detector

### TE/TM mode coupling

- TE and TM modes
- Effect of inherent coupling

**Dark pulse generation at normal dispersion**

**Effect of inherent coupling**

**Dual comb generation**


Dispersion in a cavity (spectrum domain picture)

Always need anomalous dispersion to have modulation instability gain

Resonance frequencies ($\mu$ is mode number)

$$\omega_{\mu} = \omega_0 + D_1 \mu + \frac{1}{2} D_2 \mu^2 + \cdots$$

$D_2 > 0$ : Anomalous dispersion
$D_2 < 0$ : Normal dispersion

Nonlinear Coupled Mode Equations for Kerr Comb Generation in Coupled Microcavity System

Mode coupling is the key

Dark pulse generation w/ mode coupling


- Laser sweep
- Normal dispersion
- Mode coupling
- Dark soliton

Venn diagram:
- w/o mode coupling $\kappa = 0$ (rad GHz)
- w/ mode coupling $\kappa = 3.34$ (rad GHz)

Graphs:
- Frequency domain (mode number vs power)
- Time domain (azimuthal angle vs power)
- Detuning vs intracavity power

w/o mode coupling $\mu = -3$

w/ mode coupling $\mu = -3$

✓ No comb formation
✓ 3-FSR comb & dark pulse formation
Mode coupling and MI gain

Mode coupling

\[ \Delta as = (\omega_\mu - \omega_0) - (\omega_0 - \omega_-\mu) \]

- \( \Delta as > \) Anomalous dispersion
- \( \Delta as < \) Normal dispersion

Phase matching (MI gain)


No MI region

Offset frequency \( \Delta \omega/2\pi \) (GHz)

Asymmetry factor \( \Delta as/2\pi \) (MHz)
Mode coupling for dark pulse comb generation

Deterministic FSR generation

\[ \Delta \omega / 2\pi = 5.5 \text{ (GHz)} \]

\[ \Delta \omega / 2\pi = 17 \text{ (GHz)} \]

\[ \Delta \omega / 2\pi = 36 \text{ (GHz)} \]

\[ \Delta \omega / 2\pi = 50 \text{ (GHz)} \]

\[ \Delta \omega / 2\pi = 64 \text{ (GHz)} \]

\[ \Delta \omega / 2\pi = 76 \text{ (GHz)} \]

Perfect agreement w/ experiments performed by:
Observation of noisy state

Direct transition to ML

Questions & discussions raised

- Why do we observe noisy state in normal dispersion system?
- Normal dispersion system was supposed to reach directly in a mode-locked regime

Intracavity power (a.u.)

Main

Aux.

Mode coupling for dark pulse comb generation

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Deterministic FSR generation is not always possible

Phase matching condition

- Offset frequency is at the blue shaded region:
  - 1-FSR comb generates.
  - Agrees with the observation by [Y. Liu, Optica 1 137 (2014)]

- Offset frequency is at the red shaded region:
  - strong oscillation is observed
  - Agrees with the observation by [M. Karpov, Nat. Commun., 9, 1146 (2018)]

We found that deterministic FSR generation in a normal dispersion system is not always possible even when using coupled cavities.
Kerr combs with mode coupling

Two modes coupling

CW/CCW mode coupling

TE/TM mode coupling

Dark pulse generation at normal dispersion

Effect of inherent coupling

Dual comb generation

CW-CCW comb measurement

Experimental setup

Silica toroid ($Q = 2 \times 10^7$)

Results

CW direction
Triangular envelope
Phase-locked?

CCW direction
Envelope is not smooth

CW/CCW mode coupling: effect of inherent coupling

Effect of CW/CCW coupling (experimental)


- Mode number $\mu = -2$
- $Q = 2.7 \times 10^7$
- $\kappa/2\pi = 22$MHz
- $\Gamma \approx 3.1$

In good agreement

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Effect of CW/CCW coupling (numerical)

\[ \frac{\partial A(\phi, t)}{\partial t} = -\left(\frac{\gamma}{2} + i\delta_0\right) A + i\frac{D_2}{2} \frac{\partial^2 A}{\partial \phi^2} + ig(|A|^2 + 2|B|^2)A + i\frac{\kappa_{\mu}}{2} B + \sqrt{\gamma_{\text{ext}S_{\text{in}}}} \]

\[ \frac{\partial B(\phi', t)}{\partial t} = -\left(\frac{\gamma}{2} + i\delta_0\right) B + i\frac{D_2}{2} \frac{\partial^2 B}{\partial \phi'^2} + ig(|B|^2 + 2|A|^2)B + i\frac{\kappa_{\mu}}{2} A \]

Soliton formation w/ different coupling

Weak coupling

Effect of coupling is present but, usually it is negligible

Kerr combs with mode coupling

Two modes coupling

Continuous wave (input) time

ECDL

Main

Aux.

Dark pulse (output)

Detuning = 0.004

Power (dB)

Mode number \( \mu \)

Power (u.u.)

Azimuthal angle


CW/CCW mode coupling

Mode coupling \( \kappa \)

Microcavity

Pump

Detector

CCW

CW

Detector

Intracavity power

Intracavity power

Detuning

\( \Gamma = 0.5 \)

\( P_m = 50 \) mW

\( \Gamma = 3.0 \)

\( P_m = 200 \) mW


TE/TM mode coupling

Dual comb generation

Dark pulse generation at normal dispersion

Effect of inherent coupling

**Dual-comb applications**

**Dual-comb applications: scan rate ↔ difference of repetition frequencies**

Microcombs have a potential to achieve fast scan rate due to high repetition frequencies.

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**LiDAR**

- Ultrasonic ranging
- Dual-OKS Setup
- CW Laser I
- Soliton I
- CW Laser II
- Soliton II
- Microresonator I
- Microresonator II
- Bullet 150 m/s
- Collimator
- Airgun
- Profile (mm)
- Position (1 mm/div)
- Time (10 μs/div)

![LiDAR diagram](image1)

*Science 359, 887-891 (2018)*

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**Spectroscopy**

- Absorption spectrum
- Dual-Comb Spectroscopy
- Optical Domain
- RF Domain

![Spectroscopy diagram](image2)

*Science 354, 600-603 (2016)*

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**CARS**

- Hexafluorobenzene
- Nitromethane
- Wavenumber (cm⁻¹)

![CARS diagram](image3)

*Nature 502, 355-358 (2013)*

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**CW/CCW directions**

- TE/TM mode coupling: for dual comb generation

![CW/CCW diagram](image4)

- Simple control of pump frequencies
- Small repetition rate difference

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**TE/TM modes**

- Complex control of pump frequencies
- Large repetition rate difference

![TE/TM modes diagram](image5)

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**Transverse modes**

- Microresonator
- Pump

![Transverse modes diagram](image6)
Coupled Lugiato-Lefever equations (LLEs)

\[
\frac{\partial a}{\partial t} = -\frac{\kappa(a)}{2} a + i\Delta\omega_0(a) a + i \frac{D_2(a)}{2} \frac{\partial^2 a}{\partial \phi^2} + i g(a) (|a|^2 + \sigma |b|^2) a + \sqrt{\kappa_c(a)} s_{in}(a) + \frac{\Delta D_1}{2} \frac{\partial a}{\partial \phi}
\]

\[
\frac{\partial b}{\partial t} = -\frac{\kappa(b)}{2} b + i\Delta\omega_0(b) b + i \frac{D_2(b)}{2} \frac{\partial^2 b}{\partial \phi^2} + i g(b) (|b|^2 + \sigma |a|^2) b + \sqrt{\kappa_c(b)} s_{in}(b) - \frac{\Delta D_1}{2} \frac{\partial b}{\partial \phi}
\]

(loss) (dispersion) (Kerr effects) (input) (repetition difference)

Dimensionless coupled LLEs

(Assuming \( \kappa = \kappa(a) = \kappa(b), g = g(a) = g(b) \))

\[
\frac{\partial u}{\partial \tau} = -(1 + i\alpha(u)) u + i\beta(u) \frac{\partial^2 u}{\partial \phi^2} + i(|u|^2 + \sigma |v|^2) u + F(u) + \gamma \frac{\partial u}{\partial \phi}
\]

\[
\frac{\partial v}{\partial \tau} = -(1 + i\alpha(v)) v + i\beta(v) \frac{\partial^2 v}{\partial \phi^2} + i(|v|^2 + \sigma |u|^2) v + F(v) - \gamma \frac{\partial v}{\partial \phi}
\]

\[
\tau = \frac{1}{2} \kappa t, u = \frac{2g}{\kappa} a, v = \frac{2g}{\kappa} b, \alpha(\pm) = -\frac{2\Delta\omega_0(\pm)}{\kappa}, \beta(\pm) = \frac{D_2(\pm)}{\kappa}, \gamma = \frac{\Delta D_1}{\kappa}, F(\pm) = \frac{2}{\kappa} \sqrt{\frac{2g\kappa_{c(\pm)}}{\kappa}} s_{in(\pm)}
\]

Relations, \( \alpha \): detuning, \( \beta \): second order dispersion, \( \gamma \): repetition difference, \( F \): input

TE/TM mode coupling: for dual comb generation

Soliton trapping with dimensionless coupled LLEs

\[
\frac{\partial u}{\partial \tau} = -(1 + i\alpha_u)u + i\beta_u \frac{\partial^2 u}{\partial \phi^2} + i(|u|^2 + \sigma|v|^2)u + F(u) + \gamma \frac{\partial u}{\partial \phi}
\]

\[
\frac{\partial v}{\partial \tau} = -(1 + i\alpha_v)v + i\beta_v \frac{\partial^2 v}{\partial \phi^2} + i(|v|^2 + \sigma|u|^2)v + F(v) - \gamma \frac{\partial v}{\partial \phi}
\]

\[\beta(*) = 0.01, \gamma = 0.3, F(*) = 4\]

\[\alpha \text{ is scanned}\]


\[\alpha: \text{detuning}, \beta: \text{second order dispersion}, \gamma: \text{repetition difference}, F: \text{input}\]
Soliton trapping with dimensionless coupled LLEs

\[
\frac{\partial u}{\partial \tau} = -(1 + i\alpha)u + i\beta \frac{\partial^2 u}{\partial \phi^2} + i(|u|^2 + \sigma|v|^2)u + F(u) + \gamma \frac{\partial u}{\partial \phi} \\
\frac{\partial v}{\partial \tau} = -(1 + i\alpha)\nu + i\beta \frac{\partial^2 \nu}{\partial \phi^2} + i(|\nu|^2 + \sigma|u|^2)\nu + F(\nu) - \gamma \frac{\partial \nu}{\partial \phi}
\]

\[\beta(\star) = 0.01, \gamma = 0.3, F(\star) = 4\]
\[\alpha \text{ is scanned}\]

TE/TM mode coupling: for dual comb generation

α: detuning, β: second order dispersion, γ: repetition difference, F: input
Soliton build-up

Waveforms

α: detuning, β: second order dispersion, γ: repetition difference, F: input, δ = γ(2β)^{-0.5}

Trapping conditions as functions of F and δ

Trapping conditions
(as functions of F (input) and δ (rep. difference))

Relations: α: detuning, β: second order dispersion, γ: repetition difference, F: input, \( \delta = \gamma(2\beta)^{0.5} \)

Summary

Two modes coupling

Dark pulse generation at normal dispersion

Continuous wave (input)

Main

Aux.

Detuning = 0.004

Power (dB)

Power (a.u.)

Main

Aux.

Mode number \( \mu \)

Azimuthal angle


CW/CCW mode coupling

CW(Soliton) \( P_m = 50 \) (mW)

CCW

CW(Not a soliton) \( P_m = 200 \) (mW)

CCW

Mode coupling \( \kappa \)

Pump

Detector

Detector

Effect of inherent coupling

Dual comb generation

TE

TM


Acknowledgements

The team

Support

Ministry of Education, Culture, Sports, Science, and Technology (MEXT), Japan, KAKEN #15H05429 and Q-LEAP (Quantum LEAP)