Accurate numerical modeling of a coupled cavity system and dark soliton generation

Yusuke Okabe (M2), Takumi Kato (D2), Shun Fujii (M1), Ryo Suzuki (D1)

A Kerr comb is expected to be a new optical frequency comb laser. To broaden the spectra for carrier-envelope offset control, a new dispersion control method using another cavity has been proposed and demonstrated experimentally. The experimental results revealed a new bandwidth Kerr comb, however, the theoretical study used an approximated model that did not take the second cavity into consideration. We describe the modeling of a 2-cavity system and perform a numerical simulation of dark soliton generation in a normal dispersion regime.

**Keyword**: Optical frequency comb; Microcavity device; Coupled resonators; Dark soliton

1. Background

A Kerr comb is created by generating a cascade of other spectra with a 3rd order nonlinear optical effect in a high-Q microcavity in an anomalous dispersion regime [1]. It can be made more cheaply, more compact, and with a higher repetition rate than an optical frequency comb (OFC). This means that smaller devices may be realized by replacing the OFC with a Kerr comb. However, because a microcavity has a narrow anomalous dispersion regime, we cannot obtain a Kerr comb that is mode-locked and that spans one octave [2].

Recently, to overcome this problem, research has been under way on Kerr comb generation in several normal dispersion regimes, where phase-matching cannot be satisfied [3][4]. Here, the principle is that mode-splitting induced by coupling two cavities can control the dispersion and so four-wave mixing can be realized because phase-matching is obtained. An experimental mode-locked Kerr comb produced by the above method has been reported [4] but the theoretical research used an approximated model of a one cavity system. In this research, we constructed an accurate model of a coupled cavity system by using coupled mode theory (CMT) and simulated mode-locked Kerr comb generation in a normal dispersion regime.

2. Coupled cavities

Coupled cavities are two cavities that are sufficiently close together (Fig. 1). When light is input into cavity A, some of the light moves to cavity B. Kerr comb generation is realized using this system because a Kerr comb occurs in cavity A and dispersion is controlled by cavity B. The merit of this 2-cavity system is that it offers arbitrary dispersion control by changing the size or material of cavity B.

When two cavities are coupled, the same resonant frequency will be split into two resonant frequencies (Fig. 2). This phenomenon is called mode-splitting; it changes the effective free spectral range (FSR) at the frequency so that the effective dispersion can be regarded as different from the original. Kerr comb generation requires that phase-matching be satisfied, and this can only be realized in an anomalous dispersion regime.

However, the dispersion control method enables phase-matching in even a normal dispersion regime [3][4].

![Fig. 1. Schematic model of coupled cavities. A light (yellow) input into cavity A (main cavity; red) and cavity B (auxiliary cavity; blue) controls the dispersion of cavity A. \( \kappa \) is the coupling strength.](image)

![Fig. 2. Schematic graph of mode-splitting. It shows the detuning from the original resonant frequency. The blue dotted line shows the original resonant frequency. When two resonant frequencies correspond perfectly, it splits the same two resonant frequencies (red). If the original resonant frequencies are slightly different, the split frequencies move and change the resonant dip.](image)

3. Kerr comb generation using coupled cavity system

There has been no accurate model of a coupled cavity system until now and so we constructed a model using CMT (Eq. 1). CMT can describe coupled cavities because it has equations for each resonant frequency; a nonlinear Schrödinger equation cannot describe it. Where \( \mu, \theta \) are the mode numbers (mode number 0 means the center resonant frequency nearest the input), \( A, B \) are the mode amplitudes of each cavity, \( \gamma \) is the loss (\( \gamma = \gamma_{\text{ext}} + \gamma_{\text{int}} \)), \( \delta \) is the Kronecker delta, \( \omega_{\text{in}} \) is the resonant frequency, \( \omega_{\text{in}} \) is

\[
\begin{align*}
\frac{\partial A_\mu}{\partial t} &= -\frac{\gamma_{\text{in}}}{2} A_\mu + \delta_\mu \sqrt{\gamma_{\text{ext}}} A_{0\mu} e^{i(\omega_{\text{in}}-\omega_{\text{in}})t} + i g \sum_{\alpha,\beta} A_\alpha A_\beta^* e^{i(\omega_{\alpha\beta} - \omega_{\alpha\beta})t} + i \delta_{\omega_{\alpha\beta}-\omega_{\alpha\beta}} \frac{K_0}{2} B_\beta \\
\frac{\partial B_\mu}{\partial t} &= -\frac{\gamma_{\text{in}}}{2} B_\mu + \delta_\mu \sqrt{\gamma_{\text{ext}}} B_{0\mu} e^{i(\omega_{\text{in}}-\omega_{\text{in}})t} + i g \sum_{\alpha,\beta} B_\alpha B_\beta^* e^{i(\omega_{\alpha\beta} - \omega_{\alpha\beta})t} + i \delta_{\omega_{\alpha\beta}-\omega_{\alpha\beta}} \frac{K_0}{2} A_\beta
\end{align*}
\]
the input light frequency, $g$ is the nonlinear coefficient, and $\kappa$ is the coupling strength between the two cavities.

It takes a long time to simulate these equations because they consist of several hundred differential equations. To avoid this, this simulation was performed using a fast Fourier transformation [6].

4. Kerr comb simulation

We simulated Kerr comb generation by the method described in Section 3. We used parameters from Ref. [4]. The Kerr comb was generated in a normal dispersion regime. Note that we assumed that the coupling strength was 800 MHz and that the resonant frequency and the FSR of the auxiliary cavity were 220 MHz higher, and 200 MHz smaller than those of the main cavity, respectively.

We show the simulation results in Fig. 3.

Fig. 3. Kerr comb simulation. (a) Intracavity power vs. detuning from mode 0. The blue and red lines show the intracavity powers of cavities A and B, respectively. I. Chaotic state. II. Stable but noisy. III. Mode-locked. (b) Optical spectrum of cavity A at III. The blue bar and red circles show the mode amplitudes and the phase, respectively. (c) The time waveforms of cavity A (blue) and cavity B (red) at III. The mode-locked time waveform is called a dark soliton.

In this simulation, we swept the input frequency from a higher frequency to a lower frequency. Figure 3(a) shows the intracavity power for detuning from the center frequency; I is the chaotic state, II is the static state but it had noise, III is the mode-locked state. The blue and red lines show the intracavity powers of cavities A and B, respectively. Figure 3(b) shows the optical spectrum of cavity A at III. The blue bar and red circles show the mode amplitudes and the phase, respectively. Note that when the Kerr comb is mode-locked in a normal dispersion regime, the phase becomes V-like or a slope. The soliton position determines the phase shape. Figure 3(c) shows the time waveforms of cavity A (blue) and cavity B (red) at III. The soliton is valley-shaped compared with a usual pulse, which looks like a delta function, and so the soliton is called a dark soliton (and the usual soliton is called bright soliton.) Since a dark soliton usually occurs in a normal dispersion regime (the bright soliton usually in an anomalous dispersion regime) and the shape was the same as in previous research (Ref. [4]), our model accurately describes the dynamics of Kerr comb generation with coupled cavities.

Note that there are two points arise from the experimental results in the previous research (Ref. [4]). One is that the chaotic state II did not exist and the other is that the stable state did not move to another stable state (state II to state III) in the experiment. We expect this behavior to provide new perceptions regarding the Kerr comb.

5. Conclusion

We constructed an accurate model of a coupled cavity system using CMT. We then simulated Kerr comb generation in the model with an FFT. As the result, we obtained a mode-locked Kerr comb, and a dark soliton, that matched previous experimental results. However, there are two different points. They are the presence or absence of a chaotic state and of a stable state moving to another stable state. If we reveal these dynamics, we could gain a new perception of the Kerr comb.

Reference