Numerical analysis of optical comb using Lugiato-Lefever equation with stimulated Raman scattering

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We conducted a numerical analysis of an optical comb using the Lugiato-Lefever equation and considering stimulated Raman scattering. We revealed that a transverse modes interaction via stimulated Raman scattering is closely related to the quality factor ratio of two transverse modes by comparison with experimental results. Moreover, we studied dark soliton generation via stimulated Raman scattering in a normal dispersion regime, and investigated a new method for dark soliton generation without mode coupling.

Key word: Optical comb, Stimulated Raman scattering, Lugiato-Lefever equation, Dark soliton

1. Introduction

An optical comb is a light source that has an evenly spaced spectrum generated by the Kerr effect or stimulated Raman scattering (SRS), which are nonlinear optical effects in an optical microresonator. Optical combs were first demonstrated in 2007 by T. J. Kippenberg [1]. Since then, the optical comb has been well studied both experimentally and theoretically. In particular, the Lugiato-Lefever equation (LLE) [2] is often used in numerical calculations when theoretical studies are conducted. The LLE is a nonlinear partial differential equation describing the time evolution of an electric field in an optical microresonator derived from a nonlinear Schrödinger equation, which describes the propagation of light in optical fiber and the boundary condition. The LLE cannot be solved analytically, but can be solved numerically by using the Split-Step Fourier Method (SSFM).

In this research, we conducted a numerical analysis on two systems using the LLE with SRS. The first was “Transverse mode interaction via SRS on optical comb generation”. The second was “Dark soliton generation via SRS in normal dispersion regime”.

2. Transverse mode interaction via SRS on optical comb generation

We obtained two experimental results with different spectral shapes depending on the pumping transverse mode as shown Fig. 1.

(a), (b) show spectra obtained with pumping in the high-Q transverse mode and (c), (d) show spectra obtained with pumping in the low-Q transverse mode. In (c) and (d), the comb is generated in a different transverse mode from the pump mode around 1650 nm, which is the center wavelength of the SRS gain.

Using these experimental results as a basis, we conducted a numerical analysis where we noted the quality factor of each transverse mode. The LLE with SRS is described as follows

\[ \frac{\partial E(t, \tau)}{\partial t} = \left[ \frac{\alpha_{\text{tot}}}{2} - i \delta_0 + i \sum_{k=2}^{\infty} \beta_k \left( -\frac{\partial^k}{\partial \tau^k} + i(1 - f_R) \lambda \right) |E|^{2k} \right] E + \int h_R(t') |E(t - t')|^2 \, dt' \]  

Here, \( t \) and \( \tau \) represent slow time and fast time, respectively. \( t_p, \alpha_{\text{tot}}, \delta_0, \beta_k, \gamma, \theta, \) and \( E_{\text{in}} \) represent round trip time, total cavity loss, detuning, cavity length, \( k \)th-order dispersion, nonlinear coefficient, coupled coefficient and input electric field respectively. \( f_R \) and \( h_R \) are the SRS contribution and the delayed Raman response function. We coupled the LLEs for two transverse modes to allow us to consider transverse mode interaction. Here, we assumed \( \text{TE}_{01} \) as the pump mode and \( \text{TE}_{00} \) as the interaction mode with the pump mode. We calculated the effective mode area and dispersion of each mode with the finite element method and used the results for numerical analysis. Fig. 2 shows our simulation results.

(a) shows that when \( Q_{\text{TE}_{00}} / Q_{\text{TE}_{01}} \) exceeds 2, transverse mode interaction occurs and the intracavity energy transfers from \( \text{TE}_{01} \) to \( \text{TE}_{00} \) via SRS. (b) shows the optical spectrum obtained when \( Q_{\text{TE}_{00}} / Q_{\text{TE}_{01}} = 3 \), which corresponds to our experimental value, and a comb is
generated by SRS in the TE_00 mode, which is not pumped. On the other hand, when the TE_m mode is pumped, the interaction does not occur as shown in (c). These optical spectra agree well with those in Fig. 1.

In summary, we showed that the transverse mode interaction via SRS is closely related to the quality factor of the pump mode, and the interaction occurs only when a low-Q mode is pumped.

3. Dark soliton generation via SRS in normal dispersion regime

In previous studies, dark soliton (DS) generation is demonstrated experimentally using mode coupling [3,4], but these methods are difficult and complicated. Then, we investigated a new method using the SRS for DS generation via SRS.

To conduct a steady analysis, we considered the normalized LLE as follows
\[ \frac{\partial u(t, \tau)}{\partial \tau} = \left( -1 + i \Delta - i \frac{\partial^2}{\partial \tau^2} + i |u|^2 - i T_R' \frac{\partial |u|^4}{\partial \tau} \right) u + S. \quad (2) \]
where \( \Delta, T_R' \), and \( S \) represent dimensionless values corresponding to detuning, SRS amplitude and input electric field. We assumed the boundary condition \( \tau \in [0, L'] \), \( L' = 20, S = 1.6, \) and \( T_R' = 0.1 \), and conducted a steady analysis with detuning \( \Delta \) as a parameter. Fig. 3 is a bifurcation diagram calculated from equation (2).

(a) indicates that stable DS solutions exist even when SRS is considered. DS solutions exist in the inside of bistability of a homogenous solution, and have some saddle-node bifurcations. (b) shows that the DS solutions connect with two homogeneous solutions, with the peak power and pulse width varying gradually. Then, we investigated the stability of a stable DS solution for different \( T_R' \) values. Stable DS solutions remain stable in the \( 0 \leq T_R' < 2.01122 \times 10^{-2} \) range, but become unstable in the \( T_R' \geq 2.01122 \times 10^{-2} \) range. Moreover, the position of the saddle-node bifurcation points at both ends of a stable DS solution remain unchanged in the stable range.

To investigate a method for realizing stable DS solutions with CW pumping, we conducted a numerical analysis using parameters converted from the dimensionless values used in the calculation used for Fig. 3. First, we swept the detuning \( \delta_0 \) from 0 to \( 1.582 \times 10^{-3} \), which corresponds to stable DS solutions, but the time waveform remained CW light. This result showed that SRS does not occur with the parameters of an existing stable DS solution, and so DS generation is impossible solely by sweeping the detuning. Therefore, first, we set a high pump power and performed the sweeping detuning to generate the Raman comb, and then set the pump power at the initial value as shown in Fig. 4.

(c) shows that a sweeping detuned Raman comb is generated around 1650 nm, then other combs are generated via anti-Stokes SRS, cascaded SRS, and four-wave mixing. We expected that from this state, only a single comb around the pump will remain by decreasing the pump power, but (c) indicates that all the combs disappeared. In addition to this method, we tried another method where the detuning was first swept from 0 to \( 3.0 \times 10^{-3} \), and then the pump power was decreased to the initial detuning sweeping value of \( 1.582 \times 10^{-3} \), but all the combs disappeared as in Fig. 4.

In this research, we were unable to discover a new method for DS generation, but more detailed analysis may provide a clue. Our analysis considered only a homogeneous solution. A DS solution and a route to a stable DS solution remains obscure, and so we think that the next task is to undertake an analysis with regard to a periodic solution.

References