Rigorous analysis of Kerr bistable memory in silica toroid microcavity

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We model nonlinear behavior in a silica toroidal microcavity using coupled mode theory and the finite element method to obtain Kerr bistability that does not suffer from the thermo-optic effect by optimizing the fiber-cavity coupling. Our memory might be used for quantum signal processing because we employ both ultra-high Q cavity and the Kerr effect.

Key words: Toroid cavity; Optical memory; Kerr effect; Optical bistability; Coupled mode theory;

1. Introduction

Recently, the technologies for manufacturing optical cavities have been developing. As a result, the quality factors of cavities have increased and the fabrication of ultra-high Q cavities has become easier. Formerly, it needed very high power to achieve optical bistability. But now, optical bistability can be achieved at relatively low power in ultra-high Q cavities.

Some optical memories that employ optical bistability in the cavity have already been reported [1,2]. However, most of these memories are based on the thermo-optic (TO) effect or carrier-plasma effect, and a memory based on the optical Kerr effect has yet to be achieved.

The Kerr effect can modulate the refractive index in a cavity without the absorption of light, so it is more effective than other effects such as the TO effect or carrier-plasma effect. On the other hand, it is known that the Kerr effect is very difficult to use because the refractive index change that it induces is very small. To employ the Kerr effect we have to choose materials that suppress the TO effect and the carrier-plasma effect. These materials should have a large bandgap and a small absorption coefficient and one such material is silica.

This paper proposes the use of a silica toroid microcavity which consists of silica and silicon, and describes a numerical analysis that confirms that a Kerr bistable memory is feasible in a silica toroid cavity. The analysis utilizes coupled mode theory (CMT) and the finite element method (FEM).

2. Numerical modeling

First, we describe our master equation based on the coupled mode theory in a whispering gallery mode (WGM) cavity to obtain the linear and nonlinear transmittance. The structure is shown in Fig. 1(a). Because a two-port system (a side coupled cavity with one waveguide) makes bistable operation difficult to observe, we focus on a side-coupled four-port system throughout this paper. It consists of a toroid (ring) cavity and two waveguides for input and output light. By using CMT [3] and a slow varying envelope (SVE) approximation, we obtain the master equation of the memory, which can express the time change of the envelope of the cavity mode \( A(t) \), as follows,

\[
\frac{dA(t)}{dt} = \left[ \int \frac{2\pi c}{n_0 + \Delta n} \left( \frac{1}{\lambda_0 + \Delta\lambda} - \frac{1}{\lambda} \right) - \frac{1}{2\tau_{\text{tot}}} \right] A(t) + \frac{1}{\tau_{\text{coupl}}} e^{j\theta} S_{\text{in}}(t),
\]

where \( S_{\text{in}}(t) \), \( \lambda_0 \), \( \lambda \), \( \Delta\lambda(t) \), \( n_0 \), and \( \Delta n(t) \) are the envelope of the waveguide mode in the lower waveguide shown in Fig. 1(a), the input wavelength, the resonant wavelength shift caused by the refractive index change (described later), the velocity of light, the refractive index of silica and the refractive index change in the cavity caused by nonlinear effects, respectively. \( \theta \) is the phase between the cavity mode and the waveguide mode in the lower waveguide and is expressed as,

\[
\theta = 4\pi^2 \left( n_0 + \Delta n(t) \right) (R + r) \left( \frac{1}{\lambda_0 + \Delta\lambda} - \frac{1}{\lambda} \right)
\]

Where \( R \) and \( r \) are the major and minor radii of the toroid cavity shown in Fig. 1(b), respectively. \( \tau_{\text{tot}} \) is the total photon lifetime of the cavity and is defined as,

\[
\frac{1}{\tau_{\text{tot}}} = \frac{1}{\tau_{\text{abs}}} + \frac{1}{\tau_{\text{loss}}} + \frac{1}{\tau_{\text{coupl}}} + \frac{1}{\tau_{\text{coup2}}}
\]

where \( \tau_{\text{abs}} \), \( \tau_{\text{loss}} \), \( \tau_{\text{coupl}} \) and \( \tau_{\text{coup2}} \) are photon lifetimes that correspond to the loss related to the absorption, the loss unrelated to the absorption, and the coupling between the cavity and the lower and upper waveguides, respectively. By using Eqs. (1-3), the output mode from the lower and upper waveguides \( S_{\text{out1}}(t) \) and \( S_{\text{out2}}(t) \) can be expressed, as follows,

\[
S_{\text{out1}}(t) = e^{-j\beta d} \left[ S_{\text{in}}(t) - \frac{1}{\tau_{\text{coupl}}} e^{-j\theta} A(t) \right],
\]

\[
S_{\text{out2}}(t) = e^{-j\beta d} \frac{1}{\tau_{\text{coup2}}} e^{-j\theta} A(t),
\]

where, \( \beta \) and \( d \) are the propagation constant and the length of the waveguides, respectively.

Next, we model the refractive index change caused by nonlinear effects. The carrier-plasma effect is negligible in silica due to its large bandgap, thus, only the modeling of the TO and Kerr effects is necessary. Generally the distribution of the light energy is consistent with the whispering gallery (WG) mode. Therefore, taking the spatial dependency into account, we can calculate the distribution of the refractive index changes caused by the TO \( \Delta n_{\text{TO}}(x, y, t) \) and Kerr effects \( \Delta n_{\text{Kerr}}(x, y, t) \), as
\[
\Delta n_{\text{TO}}(x, y, t) = n_0 C_{\text{TO}}(T(x, y, t) - 300),
\]
\[
\Delta n_{\text{Kerr}}(x, y, t) = \frac{2n_2 c}{n_0} \frac{U_p(t)}{2\pi R} I(x, y).
\]

Here \(T(x, y, t)\), \(n_2\), \(U_p(t) = |A(t)|^2\) and \(I(x, y)\) are cross-sectional temperature distribution, the nonlinear refractive index, the light energy stored in the cavity and the normalized cross-sectional intensity distribution obtained by the FEM [4] in advance (shown in Fig. 1(b)), respectively. Note that \(T(x, y, t)\) can also be calculated using the FEM (COMSOL Multiphysics). Now, by using Eqs. (1)-(5), we can analyze the nonlinear behavior of this optical memory.

3. Determining the photon lifetimes

Here we describe the photon lifetimes that we used in our analysis. The material absorption of silica at telecom wavelengths is usually very small (\(\alpha = 0.2\) dB/km), but, it is known that silica toroid microcavities have much larger absorption due to the water layer and contamination on the surface. And in practice the \(Q\) factor is limited by these absorptions. Thus, we decide to consider a realistic case where the experimental \(Q\) is limited by the absorption, by setting \(\tau_{\text{int}} \approx \tau_{\text{abs}} = 323\) ns (corresponding to \(Q_{\text{int}} = 4 \times 10^8\) [5]), where \(Q_{\text{int}} = \omega \tau_{\text{int}} = \omega (\tau_{\text{abs}}^{-1} + \tau_{\text{loss}}^{-1})\) is the intrinsic photon lifetime.

In general, as the TO effect is much slower and larger than the Kerr effect, the latter is cancelled out by the former after a certain time. Thus, to use the Kerr effect for the memory, we have to finish the operation before heat has accumulated significantly. The [rising and falling/rise and fall?] time of the energy in the cavity (which is directly related to the operation speed) is determined by \(\tau_{\text{tot}}\), so, we have to achieve a low \(\tau_{\text{tot}}\) in an ultra-high \(Q\) cavity such as a toroid cavity. For this, we decided to control \(\tau_{\text{coup1}}\) and \(\tau_{\text{coup2}}\). Note that \(\tau_{\text{coup1}}\) is controlled to satisfy \(\tau_{\text{coup1}}^{-1} = \tau_{\text{int}}^{-1} + \tau_{\text{coup2}}^{-1}\) and thus achieve critical coupling.

4. Results and discussions

First, we employ a rectangular pulse to obtain the refractive index change \(\Delta n_{\text{TO}}(x, y, t)\) and \(\Delta n_{\text{Kerr}}(x, y, t)\). The calculation results are shown in Fig. 2 for three different \(\tau_{\text{coup2}}\) values. The figure shows that \(\Delta n_{\text{TO}}(x, y, t)\) is larger than \(\Delta n_{\text{Kerr}}(x, y, t)\) in all three cases when \(t\) is larger than 2.3 \(\mu s\). This number gives us the upper limit of the Kerr memory holding time without suffering from the TO effect; namely it is the usable domain for Kerr memory operation, which we call "Kerr usable regime" shown in Fig. 2. Finally, Fig. 2(a) shows that the charging speed of the cavity differs for different \(\tau_{\text{coup2}}\) values. The cavity charging time is much faster for \(\tau_{\text{coup2}} = \tau_{\text{int}}/100\), which allows the cavity to reach to a plateau \(\Delta n_{\text{Kerr}}\) domain much faster. This enables us to have a longer "Kerr memory usable" regime, which allows us to use the cavity for a longer duration as an optical Kerr memory. The figure shows that we can maximize the Kerr dominant "Kerr memory usable" regime by setting \(\tau_{\text{coup2}}\) as small as possible.

Finally, the memory operation for various \(\tau_{\text{coup2}}\) values is shown in Fig. 3. Since the response speed of
depends on the total photon lifetime \( \tau_{\text{tot}} \), we normalized the temporal axis \( t \) by \( \tau_{\text{tot}} \). Figure 3 shows clearly that the reset pulse does not work when \( \tau_{\text{coup}2} = \tau_{\text{int}} \) and \( \tau_{\text{int}}/10 \), and the memory operation cannot be obtained at this condition. If the system is operating at the Kerr dominant regime, we should be able to reset the state by injecting a negative reset pulse. The required pulse width needed for the reset pulse is \( > \tau_{\text{tot}} \), since we can discharge the cavity within this time. However, the figure shows that significant heat is accumulating in the system, which makes the system impossible to reset because \( \Delta n_{\text{TO}} \) cannot be reset by such a short negative pulse due to its much longer relaxation time.

On the other hand, when \( \tau_{\text{coup}2} \leq \tau_{\text{int}}/100 \), we can successfully set and reset the system, and use the device as a Kerr bistable memory. However, the TO effect cannot be eliminated completely even in this case, and thus a holding time exists. It automatically switches \( \rho_{\text{out1}} \) from ON to OFF due to the thermal accumulation. Figure 3 shows that the memory holding time for a realistic case is about 500 ns.

\[ \rho_{\text{out1}} = |S_{\text{out1}}|^2 \]

5. Conclusion

We rigorously modeled the Kerr and TO effects in a silica toroidal microcavity by combining CMT and FEM. A clear understanding was gained of the impact on adjusting the coupling, and showed that Kerr optical bistable memory operation is possible by adjusting the coupling between the cavity and the waveguides. The memory holding time is about 500 ns. Although the driving power is 7.3 mW, the energy consumed by the system is extremely low. This is because Kerr nonlinearity does not absorb photons, while other nonlinearities such as carrier or TO effects do. In addition, due to the ultrahigh-Q of the system, the energy loss outside the system is also low. Our Kerr bistable memory in a silica toroidal microcavity has extremely low loss and thus is suitable for such applications as quantum signal processing.

References